Funkcje trygonometryczne kąta α.

$$\sin\alpha = \frac{y}{r}$$

$$\cos\alpha = \frac{x}{r}$$

$$\operatorname{tg}\alpha = \frac{y}{x}, \quad x \neq 0$$

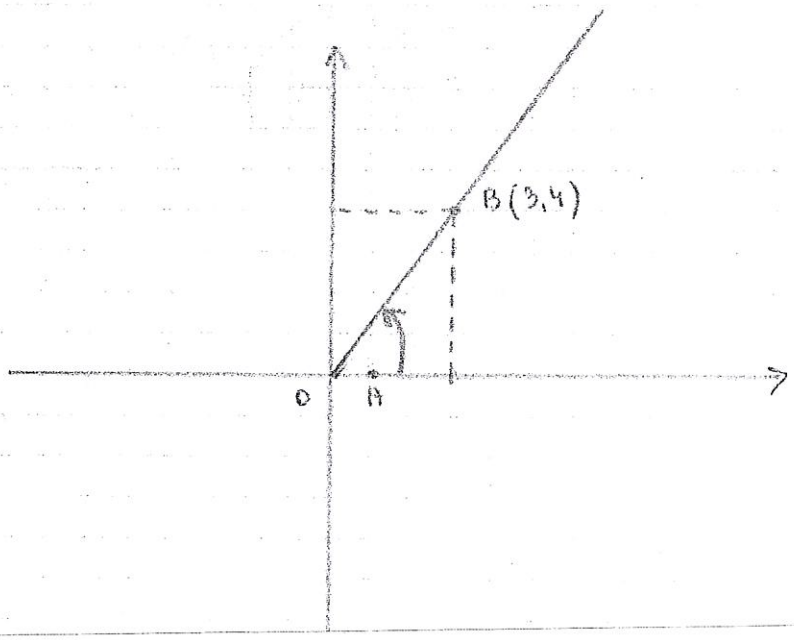
$$\operatorname{ctg}\alpha = \frac{x}{y}, \quad y \neq 0$$

1. Oblicz wartości funkcji trygonometrycznych $\sphericalangle AOB$

$A = (1,0)$ $O = (0,0)$

a) $B = (3,4)$
b) $B = (-1,2)$
c) $B = (0,4)$
d) $B = (-2,4)$
e) $B = (3,-6)$
f) $B = (0,1)$

Ad. a) $B = (\overset{x}{3}, \overset{y}{4})$



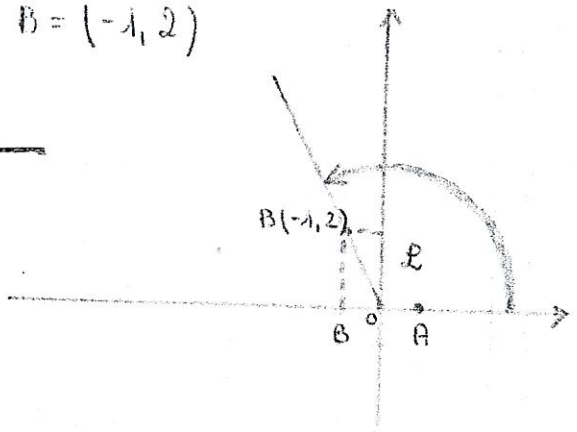
$r = |OB| = \sqrt{x^2+y^2} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$x = 3$

$y = 4$

$\sin\alpha = \frac{y}{r} = \frac{4}{5}$ $\cos\alpha = \frac{y}{r} = \frac{4}{5}$

$B = (-1, 2)$



$r = |OB| = \sqrt{(-1)^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$

$x = -1$

$y = 2$

$\sin \ell = \frac{y}{r} = \frac{2}{\sqrt{5}}$ $\quad tg\,\ell = \frac{y}{x} = \frac{2}{-1} = -2$

$\cos \ell = \frac{x}{r} = \frac{-1}{\sqrt{5}}$ $\quad ctg\,\ell = \frac{x}{y} = \frac{-1}{2}$

Ad. c

$B = (\overset{x}{0}, \overset{y}{4})$

$r = |OB| = \sqrt{0^2 + 4^2} = \sqrt{16} = 4$

$x = 0$

$y = 4$

$\sin \ell = \frac{y}{r} = \frac{4}{4} = 1$ $\quad tg\,\ell = \frac{y}{x} = \frac{4}{0}$

$\cos \ell = \frac{x}{r} = \frac{0}{4} = 0$ $\quad ctg\,\ell = \frac{x}{y} = \frac{0}{4}$

Zad. 2

2) $\sin \ell = \frac{2}{5}$ $\quad \cos \ell < 0$

- $\sin^2 \ell + \cos^2 \ell = 1$

$\left(\frac{2}{5}\right)^2 + \cos^2 \ell = 1$

$\frac{4}{25} + \cos^2 \ell = 1$

$\cos^2 \ell = 1 - \frac{4}{25}$

$\cos^2 \ell = \frac{25}{25} - \frac{4}{25}$

$\cos^2 \ell = \frac{21}{25}$

$\cos^2 \ell = \sqrt{\frac{21}{25}}$

$\cos \ell = -\frac{\sqrt{21}}{5}$

$tg\,\ell = \frac{\sin \ell}{\cos \ell} = \frac{\frac{2}{5}}{-\frac{\sqrt{21}}{5}} = \frac{2}{5} : \left(-\frac{\sqrt{21}}{5}\right) = \frac{2}{5} \cdot \left(-\frac{5}{\sqrt{21}}\right) = -\frac{2}{\sqrt{21}}$

$ctg\,\ell = \frac{\cos \ell}{\sin \ell} = \frac{-\frac{\sqrt{21}}{5}}{\frac{2}{5}} = \left(-\frac{\sqrt{21}}{5} \cdot \frac{5}{2}\right) = -\frac{\sqrt{21}}{2}$

α	30°	45°	60°
$\sin\alpha$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\alpha$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\operatorname{tg}\alpha$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\operatorname{ctg}\alpha$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

3. Związki między funkcjami trygonometrycznymi tego samego kąta.

wzór jedynkowy: $\sin^2\alpha + \cos^2\alpha = 1$

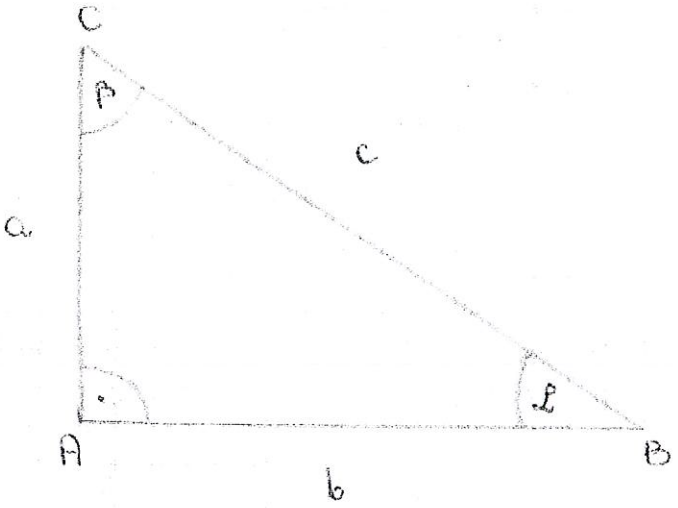
$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}$$

$$\operatorname{tg}\alpha \cdot \operatorname{ctg}\alpha = 1 \qquad \operatorname{ctg}\alpha = \frac{1}{\operatorname{tg}\alpha}$$

4. Znaki funkcji trygonometrycznych

Funkcja	I	II	III	IV
	$(0^\circ - 90^\circ)$	$(90^\circ - 180^\circ)$	$(180^\circ - 270^\circ)$	$(270^\circ - 360^\circ)$
$\sin\alpha$	+	+	−	−
$\cos\alpha$	+	−	−	+
$\operatorname{tg}\alpha$	+	−	+	−
$\operatorname{ctg}\alpha$	+	−	+	−



a) a - przyprostokątna leżąca na przeciw kąta α

b - przyprostokątna leżąca przy kącie α.

c) - przeciwprostokątna

α, β - kąty ostre

$\alpha + \beta = 90^\circ$

$\sin \alpha = \frac{a}{c}$

$\cos \alpha = \frac{b}{c}$

$\operatorname{tg} \alpha = \frac{a}{b}$

$\operatorname{ctg} \alpha = \frac{b}{a}$

$\sin \beta = \frac{b}{c}$

$\cos \beta = \frac{a}{c}$

$\operatorname{tg} \beta = \frac{b}{a}$

$\operatorname{ctg} \beta = \frac{a}{b}$

przyprostokątnych $a = 3$ $b = 4$. Oblicz wartości funkcji ostrych.

$$c^2 + b^2 = c^2$$

$$c^2 = 9 + 16$$

$$c^2 = \sqrt{25}$$

$$c = 5$$

$$\sin \alpha = \frac{a}{c} = \frac{3}{5}$$

$$\cos \alpha = \frac{b}{c} = \frac{4}{5}$$

$$\operatorname{tg} \alpha = \frac{a}{b} = \frac{3}{4}$$

$$\sin \beta = \frac{b}{c}$$

$$\cos \beta = \frac{a}{c}$$

$$\operatorname{tg} \beta = \frac{b}{a}$$

$$\operatorname{ctg} \beta = \frac{a}{b}$$